Single Dish Basics: Practicalities and Performance Parameters

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Some Important Equations

- **The antenna theorem**

  $$A_e \Omega_a = \lambda^2$$

  where Ω_a is the antenna solid angle and A_e is the effective area: $A_e = \eta_a A_p$ (η_a is the aperture efficiency and A_p is the projected area of the telescope).

- **K / Jy**
  Recall that we relate the power received by the antenna at a frequency ν, P_ν, to the antenna temperature, T_A, and the observed intensity, I_ν, as follows:

  $$\begin{aligned} P_\nu &= I_\nu A_e \Omega_a \\ 2kT_A &= S_\nu A_e \\ T_A &= \left(\frac{A_e}{2k}\right) S_\nu \end{aligned}$$

  Therefore, the conversion between K and Jy is just

  $$K/Jy = \frac{A_e}{2k}$$

The System Temperature

While the antenna temperature, T_A describes the flux received from the source you are interested in, the system temperature, T_{sys} describes the actual power received due to the sky (sources plus atmosphere) and the receiver itself (T_R).

$$T_{sys} = T_{sky} + T_R + T_{atm} + ...$$

The main component is the receiver temperature, T_R. Typically $T_R >> T_A$, so how do we detect the source we are interested in? First point the antenna at the source (on-source), then point the antenna somewhere else, preferably a part of the sky with no sources (off-source). Then,

$$T_{sys}(on - source) = T_A + T_R + T_{atm} + ...$$

$$T_{sys}(off - source) = T_{blank\ sky} + T_R + T_{atm} + ...$$

So in order to detect the source, we need $T_A > \Delta T_{sys}$ (the noise in our measurement of T_{sys}). By 'Root-N statistics', the noise is just $T_{sys}/\sqrt{N}$ where N is the number of independent samples of the signal. The number of independent samples is $\Delta\nu \cdot \tau$, where $\Delta\nu$ is the bandwidth (Hz) and τ is the integration time (seconds). With a bandwidth $\Delta\nu$, the signal is statistically independent over a time interval $1/\Delta\nu$, so that the number of independent samples is just τ divided by $1/\Delta\nu$, so $N = \tau\Delta\nu$. Therefore,

$$\Delta T_{sys} = \frac{T_{sys}}{\sqrt{\tau\Delta\nu}}$$

The Radiometer Equation

Now we can write down an expression for the signal to noise ratio (the radiometer equation):

$$\frac{S}{N} = \frac{T_A}{\Delta T_{sys}} = \frac{T_A}{T_{sys}}\sqrt{\tau \Delta \nu}$$

Typical values might be $\Delta\nu = 10$ MHz, $\tau = 1$ sec so that $\sqrt{\tau\Delta\nu} \sim 3 \times 10^3$. Typical values for T_{sys} are 40 - 200 K.

SEFD

The SEFD is the 'system equivalent flux density'. This is basically the flux equivalent of T_{sys}:

$$SEFD = \frac{T_{sys}}{(K/Jy)} = \frac{T_{sys}}{A_e/2k} = \frac{2kT_{sys}}{A_e}$$

The SEFD is a useful way to compare the sensitivity of two different systems since it folds in both T_{sys} and A_e. This also greatly simplifies the sensitivity calculation: if you know the flux in Jy of the source you want to detect, and you know the SEFD, then you can easily calculate the integration time you need to make a given S/N detection (for an unresolved source!):

$$\frac{S}{N} = \frac{S_\nu(Jy)}{SEFD}\sqrt{\tau\Delta\nu}$$

Real Antennas

Ideally, we would like to point our antenna at a source and receive all the flux from that source and only the flux from that source. This is not the case. First, recall that when you are thinking about antennas, the Reciprocity Theorem applies: the antenna may act as a receiver or transmitter - they are equivalent.

Considerations for Real Antennas:

- Diffraction
  - The circular aperture of the antenna results in a diffraction pattern like an Airy disk. This pattern is the 'beam' of the antenna. The main lobe of the beam receives most of the power. The FWHM of the main lobe is what is referred to when talking about the resolution of the telescope. The sidelobes are the rest of the beam pattern - these are receive less power but can be a problem if you have a strong source in a sidelobe. The backwards-pointing sidelobe is the 'Arago spot'.
  - You also get diffraction around support structures and feed legs (one reason why unblocked apertures like the GBT and ATA are good).
- Ohmic losses: never perfect reflection - always some absorption. These are typically much less than 1% but multiple reflections in a system can add up to become important.
- Blockage by feed legs and support structure (another reason unblocked apertures are nice!)
- Scattering: radiation from the ground can scatter into the feed off the support structures.

- Error beam response (aka Ruze loss): this is due to surface deviations from the theoretical shape of the dish. The forward power from these surface irregularities results in a broad error pattern (see Figure 5 in Millimeter Wave Calibration Techniques). Generally, you are in good shape if the rms deviations are smaller than $\lambda/20$.

- Illumination losses: spillover (rear and forward) can allow power from the sky or the ground to get into the system (see Figures 6 - 9 in Millimeter Wave Calibration Techniques).

Note: Some of these losses (especially illumination losses) are said to either terminate at the ambient temperature or the sky temperature. This is important because in general, things that terminate at the ambient temperature (300 K!) will give you much more unwanted power than the sky temperature. However, spillover that terminates at sky temperature can be very important for galactic atomic hydrogen (HI) studies, since there is HI emission over the whole sky, spillover could give you a lot of power from a region of the sky outside of the region of interest.

Confusion

Confusion refers to the case of having multiple sources blended together inside of the beam (see Figure 7 in Continuum 1: General Aspects). With knowledge of the number of sources as a function of brightness at a given frequency, one can estimate (for a given beam size) the probability that an observed source of some brightness is actually composed of multiple sources. The figure of merit here is the 'rms confusion': generally one should only trust sources brighter than 5 times the rms confusion.

Sidenote: 'natural confusion' is a problem expected for SKA. As higher redshift galaxies become visible, their angular size and large numbers could mean that the sources actually overlap. This is discussed for nanoJy sources.

'1/f Noise'

This refers to the variations in the gain and system temperature of the antenna with time. These can be intrinsic to the system as well as a result of atmospheric fluctuations (mostly due to water vapor, which becomes increasingly important at higher frequencies). The resolution of this is to use short integration times and later calibrate out these fluctuations. This will be discussed in more detail in the calibration discussion.